Dynamic Signature for a Closed-Set Identification based on Nonlinear Analysis

David Ahmedt-Aristizábal, Edilson Delgado-Trejos, Jesús Francisco Vargas-Bonilla and Jorge Alberto Jaramillo-Garzón

Abstract

This paper presents a study of biometric identification using a methodology based on complexity measures. The identification system designed, implemented and evaluated uses nonlinear dynamic techniques such as Lempel-Ziv Complexity, the Largest Lyapunov Exponent, Hurst Exponent, Correlation Dimension, Shannon Entropy and Kolmogorov Entropy to characterize the process and capture the intrinsic dynamics of the user’s signature. In the validation process 3 databases were used SVC, MCYT and our own (ITMMS-01) obtaining closed-set identification performances of 98.12%, 97.38% and 99.50% accordingly. Satisfactory results were achieved with a conventional linear classifier spending a minimum computational cost.

1. Introduction

The most recent technology for automatic human recognition is biometrics, which can be used for identification or verification [1]. Automatic identification systems are a major concern in this age of automation, given that they play an important role in security. Therefore, biometrics is considered to be a secure and convenient authentication tool since it cannot be borrowed, stolen or even forgotten [1]. The signature identification is based on the assumption that the signature of each individual is unique and can be associated with a specific pattern. Among biometrics, signature presents some advantages: wide acceptance, common use in legal and commercial transactions and usage of relatively cheap devices [2, 3]. However, while physiological biometrics can be adequately represented by a single sample, behavioral biometrics (such as signature) usually requires several samples due to its inherent variability [4]. These variations may originate from a wide number of factors and are person dependent. Therefore, the need arises for automatic recognition using techniques that can reliably interpret the physiological behavior of the signer.

Psychomotor skills have been found to be controlled by quite complex dynamical systems whose properties and structures are yet to be fully described and understood [5]. Psychomotor actions found in handwriting are a product of a chaotic dynamic process where the initial conditions depend on the environmental and biomechanical context [5]. Keeping this in mind, it is necessary to highlight the existence of many complex physiological processes that have been interpreted as reflecting the chaotic states [6]. Therefore, it is possible to assume that the theoretical performance analysis of nonlinear dynamic systems can provide new and more powerful concepts for analyzing the real behavior of complex systems like the biological ones [7].

There have been numerous techniques for signature authentication [8]. In particular, elastic matching using Dynamic Time Warping (DTW) [9] is the most widely studied on-line signature authentication technique, while Hidden Markov Models (HMMs) [10] have become the best-performing statistical models for on-line signature verification [1]. Nevertheless, DTW is computationally expensive and the resampling process usually results in losing important local details so that forged signatures closely match genuine ones. In turn, due to the statistical nature of HMM the number of training data is usually quite high for practical applications.

Recent proposals show some improvements by combining techniques [2], fusing local and global information [11] or by combining different system outputs [10]. However, excepting some isolated cases, the high number of used features leads to greater computational demand as well as increased storage needs. Furthermore, the higher the complexity of the features, the higher the acquisition cost of the device.

As the biometric information of the signature contains embedded nonlinear operators, the use of nonlinear dynamic analysis techniques should provide a more reliable performance of biometric identification. This paper presents a methodology for closed-set identification from a feature extraction procedure based on complexity measures. In order to evaluate the performance of the computed features space, a leave-one-out cross-validation was performed with two commonly used classifiers: k-nearest neighbors and linear discriminant classifier.
2. Materials and Methods

2.1. Database

Three signature databases have been employed in our experiments: SVC, MCyT and our own signature database. In this study, only one type of experiments was performed: closed-set identification. Therefore, for each database, the genuine sample for each user was employed. Each signature from the databases was characterized by X and Y coordinates, pressure, azimuth, and altitude information.

ITMMS01 Database. This is our own signature database (only genuine signatures), called ITM-MIRP-SIGN-01 (ITMMS01) in gratitude to the university and the research group. It contains 800 signatures gathered from 40 different volunteers. Among those subjects, there were 23 females, two left-handed and ages vary between 20 and 50. Each subject was asked to contribute 20 signatures collected in two sessions. There was approximately two weeks time between the sessions. Ten signatures were collected from each subject during each session. There were no constraints, so the subjects signed in their most natural way; in an arbitrary orientation. Therefore, there was a significant intra-class deformation and variation among signatures that belong to the same subject. The signatures were collected with a digitizing tablet (WACOM Intuos 4), without visual feedback. We designed a mechanism which was used by the authors to interface the biometric sensors using .NET Platform and Component Object Model (COM) as an interface for the device.

SVC Database. This is a publicly available handwritten signature provided by the First International Signature Verification Competition (SVC2004) [12], which consist of 20 authentic signatures per person gathered from 40 people.

MCyT Database. This study also used a subset of the complete MCyT Signature Database Corpus of 96 users with 25 samples each [13]. This database is larger than those typically used in the literature.

2.2. Biometric identification

The focus of this work was closed-set identification, which is a biometric task where an unidentified individual is known to be in the database and the system attempts to determine his/her identity. The primary method of assessing the performance of a closed-set identification system is the Identification Rate, the rate at which an individual in a database is correctly identified. In addition, the cumulative match curve (CMC) is used as a measure of 1:m identification system performance. It judges the ranking capabilities of an identification system. The receiver operating characteristic curve (ROC) of a verification system, on the other hand, expresses the quality of a 1:1 matcher [14]. Also this work deals with on-line (dynamic) signature identification, which means that the system tracks down trajectory and other time-sequence variables using specially designed tablets or other devices during the act of signing.

2.3. Nonlinear analysis

The nonlinear time series analysis constitutes an alternative mathematics for analyzing discrete time signals and physiological signals of human body [15, 16]. For physical processes such signatures which are commonly dynamic, or time dependent, it is almost always the case that we do not have access to the actual equations governing a system. Instead, we have a set of observations called time series. Changes over time in the state of the system, represented by the model’s solution, can be represented graphically as a plot of key variables such as velocity and position solutions called the state, or phase, space. Over time the solution of a dynamic process moves through this state space along one of its trajectories. An examination of the state space can inform us about the overall behavior of the system. For example the output of a process often contains equilibrium regions to which solutions of the system converge or are attracted [17]. There is an attractor, called a chaotic or strange attractor, in which the phase trajectories are confined to a finite space, but local points diverge. This is described as stretching and folding of trajectories. Chaotic systems are characterized by complex solutions and dynamics, but are the result of simple deterministic equations. The output of a chaotic system is so complex that it appears random but has some short term predictability [18].

Embedding. The nonlinear dynamic analysis requires a reconstruction phase in the state space of the signal studied. This analysis supports the understanding of the system dynamics variables and the evolution over time of the same. This process is known as embedding, and requires the choice of certain parameters required for a correct reconstruction, to improve the observability of the trajectories that the system states evolve.

A plot of important systems variables can tell us a lot about that system’s dynamics. However, in experimental time series we do not have access to the various variables we may wish to plot. What is needed is a way of reconstructing the dynamics of the system from observables. One solution developed in [19, 20, 21], is to generate a topological equivalent to the state vector \( \mathbf{x}[n] \), by taking the observable \( x[n] \) as the first coordinate, \( x[n+\tau] \) as the second and the \( x[n+(d-1)\tau] \) as the last. \( \tau \) is the delay parameter and \( d \) is the embedding dimension.

\[
\mathbf{x}[n] = \{x[n], x[n+\tau], x[n+2\tau], ..., x[n+(d-1)\tau]\} \tag{1}
\]
The embedding lag should be chosen so that each successive point is independent from previous points, for example by making sure they are decorrelated [19]. The most popular methods for finding a delay value to achieve an accurate and practical embedding are based on the first minimum of mutual information [22] and on the first zero of the autocorrelation function [23]. The effect of this embedding technique is to transform the unidimensional time series into a sequence of vectors in a d-dimensional space. To achieve accurate results, the embedding dimension, d, needs to be chosen so that \( d \geq 2m + 1 \), where m is the dimension of the underlying attractor. The result of this method is known as a delay plot. It has been demonstrated that if suitable embedding dimensions and lags are used, this technique is mathematically equivalent to what would have been achieved when plotting the known system variables [19].

**Fractal features.** The nonlinear dynamics can be quantified by reconstruction of an attractor containing the intrinsic variability of the system. After reconstruction with the embedding techniques are applicable to the feature extraction using typical complexity measures as is shown in Figure 1. The subset of the feature vector that characterizes the chaos-based features is calculated for each time series from each signature, so each series is described by a six-dimensional vector that contains the fractal features. Software tools like TISEAN and TSTOOL were used for the calculation of the features. A brief description of the fractal features is given next.

**Lempel-Ziv Complexity (LZ).** Lempel and Ziv proposed a useful complexity measure, which can characterize the degree of order or disorder and development of spatiotemporal patterns [24]. The calculation of the Lempel-Ziv complexity, estimates the complexity or irregularity of a time series and is usually calculated using the algorithm described in [25]. The values range from near 0 to 1. Particularly, LZ = 1 means maximum complexity (totally random pattern white noise), and LZ = 0 means perfectly predictable series (deterministic equation).

**Largest Lyapunov Exponent-LLE (\( \lambda_1 \)).** For a dynamical system, sensitivity to initial conditions is quantified by the Lyapunov exponents. There is a Lyapunov exponent for each dimension of the process, which together constitute the Lyapunov spectrum for the dynamical system. For example, consider two trajectories with near initial conditions on an attracting manifold. When the attractor is chaotic, the trajectories diverge, on average, at an exponential rate characterized by the LLE [26]. So the Lyapunov exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories. A positive LLE is usually taken as an indication that the system is chaotic [23]. Small deviations of both trajectories, \( \hat{\xi}_x[n] \) and \( \hat{\xi}_y[n] \) can be given by:

\[
u[n + 1] = J \{ \hat{\xi}_x[n] \} \nu[n] \quad (2)
\]

where \( J \{ \hat{\xi}_x[n] \} \) is the Jacobian matrix, evaluated for a reference point of \( \hat{\xi}_x[n] \). If \( n_0 \) is the initial sample and \( u[n_0 + \Delta n] \) is the distance between the \( \hat{\xi}_x[n] \) and \( \hat{\xi}_y[n] \) trajectories after \( \Delta n \) sampling periods, then to measure the exponential separation of the trajectories it is assumed that in the future distant (\( \Delta n \gg 0 \)), the norm of the \( u[\Delta n] \) vector behaves like [23]:

\[
u[n_0 + \Delta n] = \| u[n_0] \| e^{\lambda_1 \Delta n} \quad \forall \chi \in \mathbb{R}, u[n_0] \ll 1, \Delta n \gg 1
\]

**Hurst Exponent (H).** The Hurst exponent expresses the correlation between different points within the time series. This parameter determines whether a time series can be represented as Brownian motion. If H exists, its value ranges from 0 to 1, showing nonlinear behavior of the time series [27]. Particularly, H = 0 means Brownian motion, 0<H<0.5 means that high-frequency terms are contained in the time series, so previous tendencies tend to be reversed in the future. Lastly, 0.5<H<1 means a soft dynamic of the time series (previous tendencies persist in the future). The calculation of the Hurst exponent of the state space \( \hat{\xi}[n] \) is obtained by the following empirical regression as the slope of the ratio,

\[
R / \sigma = (\tau / 2)^h \quad (4)
\]

where R is the span variation (difference between maximum value and minimum value in the \( \hat{\xi}[n] \) series), \( \sigma \) is the standard deviation, and \( \tau \) is the delay used in the reconstruction of the attractor.
Correlation Dimension \((D_2)\). The dimension of a system refers to the minimum number of scalar variables needed to model the dynamic process, or contain the attractor and hence provides a measure of the system’s complexity. A commonly employed measure is the correlation dimension, \(D_2\), which is based on geometric properties of the attractor in phase space. \(D_2\) is a measure of the dimensionality of the space occupied by a set of random points, often referred to as a type of fractal dimension, and gives a quantity of the nature of the attractor trajectory [28]. Specifically, \(D_2\) value of the attractor is calculated as follows [23]:

\[
D_2 = \lim_{r \to 0} \frac{\log(C(r))}{\log(r)} \quad (5)
\]

where \(C(r)\) is the correlation integral:

\[
C(r) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \Theta \left( r - |\vec{z}_i - \vec{z}_j| \right) \quad (6)
\]

where \(\vec{z}_i\) and \(\vec{z}_j\) are the points of the trajectory in the phase space, \(r\) is the radial distance to each reference point \(\vec{z}_i\), and \(N\) is the number of points for a time series in a \(d\)-dimensional space. Notation \(\Theta\) stands for Heaviside function.

Shannon Entropy, the Shannon entropy is of fundamental importance in communication engineering and coding theory. It measures the uncertainty over the expected state of a system. Moreover, it is a measure of the surprise associated with a particular outcome \(x\) [29]. The Shannon entropy is defined as follow:

\[
H(X) = \sum_{k} f(p(X)) \quad (7)
\]

and the expression \(f(p(X))\) is calculated as follow:

\[
f(p(X)) = - p(X) \log(p(X)) \quad 0 \leq p(X) \leq 1 \quad (8)
\]

where \(p(X)\) is the probability that a random variable \(X\) takes the value of \(k\), with \(k=1, 2, \ldots, N\).

Kolmogorov Entropy, the Kolmogorov-Sinai entropy, also known as the metric entropy or simply the Kolmogorov entropy, is defined as a measure of the loss of information along the system evolution. It is a measure of the uncertainty associated with a randomly selected trajectory [30]. Specifically, \(K_2\) value of the attractor is calculated as follows

\[
K = - \lim_{r \to 0} \frac{1}{m} \sum_{p(k_1, k_2, \ldots, k_m)} p(k_1, k_2, \ldots, k_m) \ln p(k_1, k_2, \ldots, k_m) \quad (9)
\]

\[
K_{2,m}(r) = - \frac{1}{m} \log \frac{C_{m,r}(r)}{C_{m,r}(1)} \quad 0 \leq K_{2} \leq K \quad (9)
\]

where \(m\) is the dimension of immersion, \(r\) is the delay value, and \(C_{m,r}(r)\) is the correlation integral defined in the previous calculation of the correlation dimension. \(k_2 \to \infty\) for random systems, while for chaotic systems \(0 < k_2 < \infty\). Kolmogorov used as an indicator of chaos the average loss information that takes place as this evolves over time. For 1D systems, \(K = 0\) (no loss of data, regular path), \(K > 0\) (loss of information, chaotic trajectories). The Kolmogorov entropy is also defined as the sum of Lyapunov exponents as follow [29]:

\[
K = \sum_{\lambda_i > 0} \lambda_i \quad (10)
\]

2.4. Proposed procedure

The representation space was composed of features estimated by using nonlinear dynamic techniques applied to each series. The data from each signature were split into the time series of pressure, azimuth, and altitude, the horizontal (\(X\)) and vertical (\(Y\)) direction. The basic velocity profiles were calculated by taking the first difference of the positional data. Each series is described by a six-dimensional vector that contains the fractal features and a final feature vector is built up for each signature concatenating all the features extracted. Table 1 summarizes the entire feature set estimated for each signature.

<table>
<thead>
<tr>
<th>Family of Parameters</th>
<th>Description</th>
<th>Series</th>
<th>Number of Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractal Analysis</td>
<td>Largest Lyapunov Exponent</td>
<td>X-Coordinate</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>Hurst Exponent</td>
<td>Y-Coordinate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correlation Dimension</td>
<td>Velocity X-Coordinate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lempel-Ziv Complexity</td>
<td>Velocity Y-Coordinate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shannon Entropy</td>
<td>Azimuth</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kolmogorov Entropy</td>
<td>Altitude</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fractal Analysis</td>
<td>Pressure</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Description of the feature set extracted for each signature from the time series.

Our approach is divided into three main phases: Feature extraction, dimensionality reduction and detection. Figure 2 depicts the overall framework of our proposed system. The fractal features are based on nonlinear dynamics and they are able to quantify the nonlinear behavior of the signature. The use of fractal features is motivated based on the fact that the system dynamics (including the nonstationary behavior) are embedded intrinsically into the attractor, and the measure of complexity in the reconstructed trajectory is able to characterize the dynamics.
As explained before, the feature set matrix $X$, was composed by the fractal features of each signature. The dimension of the matrix $X$ was $q \times s$, $q$ being the overall number of samples for all users ($q = 2400$ for MCyT; $q = 800$ for SVC and ITMMS01), and $s$ the number of features extracted ($s = 42$). The discriminative power of the features in the reference set plays a major role in the whole identification process, as it is important to find features that do not vary with small changes in intra-class signatures, yet that are powerful enough to be used to discriminate other signature classes. With the aim of finding a feature subset that minimizes the classification error, a Sequential Forward Selection Algorithm (SFS) [31] with a cost function based on 1-Nearest Neighbor leave-one-out [32] was used to select the most significant features. Validation of classification procedures was achieved using a leave-one-out cross-validation strategy [33]. Conventional classifiers such as the Linear Normal Bayes Classifier (lnc) and the nonlinear k-Nearest Neighbor Classifier (kmnc) were applied [31]. The performance of the closed-set identification is represented by the curve Cumulative Match Characteristic (CMC).

### 3. Results and Discussion

The closed-set identification results are represented in the percentage of signatures that were correctly assigned to each user (Identification Rate), so for the databases mentioned above we obtained performances of 98.12%, 97.38% and 99.50% accordingly. Table 2 shows the summary of the results of these experiments. It clearly demonstrates the suitability and superiority of using the proposed Nonlinear Analysis approach for feature extraction in online signature identification. The purpose of feature reduction is to identify the significant features and eliminate the irrelevant or dispensable features from the learning task. The benefits of feature reduction are twofold: firstly, it considerably decreased the computation time and secondly, it increases the performance of the resulting classification. Sequential Forward Selection has been employed here to remove redundant conditional attributes from discrete-valued datasets, while retaining their information content. This approach has been applied to aid classification of online signatures, with very promising results.

<table>
<thead>
<tr>
<th>Database</th>
<th>Total Number of Signatures</th>
<th>Number of Signatures per Subject</th>
<th>Number of Feature Reduction</th>
<th>Identification Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVC</td>
<td>800</td>
<td>20</td>
<td>29</td>
<td>98.12%</td>
</tr>
<tr>
<td>MCYT-96</td>
<td>2400</td>
<td>25</td>
<td>16</td>
<td>97.38%</td>
</tr>
<tr>
<td>ITMMS01</td>
<td>800</td>
<td>20</td>
<td>25</td>
<td>99.50%</td>
</tr>
</tbody>
</table>

Table 2. Summary of results with leave-one-out cross validation.

Nonlinear dynamic techniques such Lempel-Ziv complexity and Kolmogorov entropy are more representative. Figure 3 illustrates that class differences were significant in the X coordinate of the signature using Lempel-Ziv complexity. Each class generates one cluster that differentiates it from the rest.

Figure 3. Separation of classes with Lempel-Ziv complexity feature in the MCyT database.

The Nonlinear algorithms that were used measured, in general, two aspects of the signature: the degree of entropy or “predictability” of the system (Kolmogorov entropy) and the minimum number of variables, components or generators that help describe the behavior of that system (Lempel-Ziv complexity), that is, the two most important measures of complexity for the 3 databases. Finally the cumulative match characteristic curve (CMC) was used as a measure of 1:m identification system performance. We analyzed the behavior of the CMC curves for the 2 classifiers that achieved better performance in the selection...
process of the classifier for the validation strategy. For instance, Figure 4 shows the performance of the two classifiers in the largest databases and Table 3 shows the results of the area under the curve and the computational cost of each classifier in each database. For each of the curves this area is calculated using numerical integration methods based on the trapezoidal rule. The computational cost is considered as the execution time of the cross-validation strategy with only one repetition for each of the databases. The implementation of the algorithms was carried out on the same processor Intel® Core™ 2 CPU, T5300@1.73GHz, 794MHz, 1.99GB of RAM with Windows XP, Service Pack 2.

<table>
<thead>
<tr>
<th>Database</th>
<th>Maximum Area</th>
<th>Classifier</th>
<th>Identification Rate (%)</th>
<th>Area under the curve</th>
<th>Computational Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVC</td>
<td>40</td>
<td>ldc</td>
<td>98.12%</td>
<td>97.25%</td>
<td>14 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>knnc</td>
<td>98.06%</td>
<td>97.86%</td>
<td>28 min</td>
</tr>
<tr>
<td>MCYT-96</td>
<td>96</td>
<td>ldc</td>
<td>97.38%</td>
<td>98.91%</td>
<td>66 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>knnc</td>
<td>97.92%</td>
<td>98.92%</td>
<td>1680 min</td>
</tr>
<tr>
<td>ITMMS01</td>
<td>40</td>
<td>ldc</td>
<td>99.50%</td>
<td>97.39%</td>
<td>12 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>knnc</td>
<td>99.56%</td>
<td>97.26%</td>
<td>25 min</td>
</tr>
</tbody>
</table>

Table 3. Summary of the comparative performances of the classifiers.

It should be clarified that with such close identification results, the difference in the area under the curve with the ldc classifier is not very significant, and also a small gain in the computational cost is too much for the knnc classifier, about 15 times greater in the case of the MCYT database. The findings reported in this paper show that the features based on nonlinear dynamic techniques, represent sufficient separability between classes after a proper tuning process of the classifiers. Figure 5 illustrates the performance for partially divided sets of the selected fractal features with the Linear Bayes classifier. This performed representation was computed by including the whole set of relevant features, which were computed by adding the features one by one ordered in decreasingly relevance. In addition, detailed observation of Figure 4 clarifies that after a certain number of added features the assessed performance tends to an asymptotical value. In this training case, when taking a partially divided set, the best performance value achieved was 97.38%, if using the ldc classifier with the subsets of the MCYT database (96 subjects). This study has found that the linear classifier is sufficient for verifying the validity of the chosen features, due to its simplicity and sturdiness.

The evaluation technique used to obtain the performance of biometric identification was the leave-one-out cross validation (LOOCV). However, for experimental reasons we used the bootstrapping method, to analyze the potential of the fractal feature space achieved for each of the databases used. For these tests the Linear Bayes classifier (ldc) was also used with same number of feature and total number of signature from each database. In this case, the percentage of samples for training was changed. For each percentage of selected training a specific number of tests is generated, which is able to diversify the split of all samples for training and testing. In each iteration a proportion of samples is randomly selected (without replacement) to train the classifier. Table 4 summarizes the results using the validation method mentioned above. It may be concluded that the identification rate was acceptable, efficient and similar to the results in Table 2 for 50% of the training samples. As the percentage of samples decreases, so does the error gap but the error average increases into very small scales. However with this method it is possible that subsets of the experiments can overlap, unlike the methodology applied to cross-validation where all samples are used for training and testing. In Figure 6 shows the consistency of classification in the scope of error achieved for each test with the MCyT database. In this database for 50% of the training samples (13 samples), the error oscillates in a small gap of 0.02. This means that the fractal analysis is strong and there is no presence of noise. Thus, the feature space is made up of relevant features and not by those that are irrelevant or redundant. The system requires a small number of user signatures to achieve a high performance.

<table>
<thead>
<tr>
<th>Database</th>
<th>Number of Tests</th>
<th>Training (%)</th>
<th>Error Gap</th>
<th>Error Average</th>
<th>Identification Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVC</td>
<td>50</td>
<td>90.0%</td>
<td>0.05</td>
<td>0.0172</td>
<td>98.28%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70.0%</td>
<td>0.0417</td>
<td>0.0204</td>
<td>97.86%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50.0%</td>
<td>0.0275</td>
<td>0.0267</td>
<td>97.33%</td>
</tr>
<tr>
<td>MCYT-96</td>
<td>100</td>
<td>90.0%</td>
<td>0.0521</td>
<td>0.0267</td>
<td>97.33%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70.0%</td>
<td>0.0238</td>
<td>0.0272</td>
<td>97.28%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50.0%</td>
<td>0.02</td>
<td>0.0278</td>
<td>97.22%</td>
</tr>
<tr>
<td>ITMMS01</td>
<td>50</td>
<td>90.0%</td>
<td>0.0167</td>
<td>0.0069</td>
<td>99.40%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>70.0%</td>
<td>0.0125</td>
<td>0.0074</td>
<td>99.26%</td>
</tr>
</tbody>
</table>

Table 4. Summary of results with bootstrapping validation.
Figure 6. Identification rate performance by bootstrapping validation.

It is difficult to compare the performance of different signature identification systems since each system uses its own signature dataset. The lack of a standard international signature database is still a big problem for performance comparison. For the sake of completeness, Table 5 presents the results obtained with the proposed approach in this study compared to other published methods.

Table 5. Comparison of proposed approach with other published methods.

<table>
<thead>
<tr>
<th>Database</th>
<th>Number of Users</th>
<th>Number of Signatures per Subject</th>
<th>Identification Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-Mayyan et al. [34]</td>
<td>OWN</td>
<td>116</td>
<td>96.4%</td>
</tr>
<tr>
<td></td>
<td>108</td>
<td>20</td>
<td>99.50%</td>
</tr>
<tr>
<td></td>
<td>10 (1st session)</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>108</td>
<td>10 (2nd session)</td>
<td>98.00%</td>
</tr>
<tr>
<td>Vivaracho-Pascual et al.</td>
<td>MCYT</td>
<td>210</td>
<td>96.4%</td>
</tr>
<tr>
<td></td>
<td>230</td>
<td>25</td>
<td>99.1%</td>
</tr>
<tr>
<td>Pascual-Gaspar et al. [4]</td>
<td>MCYT</td>
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<td>95.78%</td>
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<td></td>
<td>SVC</td>
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<td>10 (1st session)</td>
<td>98.12%</td>
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<td>Proposed</td>
<td>MCYT</td>
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<td>UTRR-01</td>
<td>40</td>
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5. References


